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LINEAR VISUOMOTOR TRANSFORMATIONS IN MIDBRAIN SUPERIOR COLLICULUS CONTROL SACCADIC EYE-MOVEMENTS

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It is well established that a localized population of neurons in the motor map of the midbrain superior colliculus (SC) drives a saccadic eye-head gaze shift. However, there is controversy as to how the brainstem saccade burst generators decode the SC activity. We focus on eyemovement generation by comparing two competing schemes from the recent literature that are both supported by neurophysiological evidence: the vector-averaging scheme versus the vector summation model. Whereas the former contains at least four nonlinearities to explain visuomotor planning and saccade execution, the latter relies predominantly on linear operations. We have demonstrated that the summation model accounts for the nonlinear main sequence of saccade kinematics, and predicted that this results from a spatial gradient in temporal burst profiles of SC cells: rostral cells have higher peak-firing rates and shorter burst durations than caudal cells. Yet, the number of spikes in their saccade-related bursts is identical. In contrast, the averaging model does not predict such activity profiles. We now also show that by incorporating the concept of predictive remapping in the spatial updating of saccade sequences, the phenomenon of target averaging in double-stimulation experiments, and the occurrence of goal-directed, but highly curved saccades in the double-step paradigm, can all be explained by the same linear summation mechanism. We argue that the linear model is more in line with neurophysiological data, while relying on fewer ad-hoc assumptions than the nonlinear vector-averaging scheme.

Keywords: Saccades; superior colliculus; main sequence; linearity; nonlinearity; models; monkey; population coding; optimal control.

1. Introduction

1.1. Saccade kinematics: A nonlinear system

Saccadic eye movements ("saccades", for short) are the rapid, conjugate displacements of the eyes that have the function to redirect the fovea as fast and as accurately as possible to a peripheral target of interest. Visually evoked saccades have remarkably stereotyped movement characteristics: their trajectories are virtually

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Fig. 1. Properties of saccade kinematics. (a) The main sequence betrays a nonlinearity (NL), as for a linear system (L) saccade duration would be constant, and peak eye velocity should increase linearly with amplitude. (b) The shape of velocity profiles depends on saccade duration: large saccades are more skewed than small saccades. (c) Component stretching of oblique saccades: their horizontal and vertical velocity components have the same shape and duration (adapted from [48]).

straight, there is a consistent straight-line relationship between movement duration and saccade amplitude, the saccade acceleration time is nearly constant for all saccades, and the relationship between peak eye velocity versus saccade amplitude is described by a saturating function (Figs. 1(a) and 1(b)).

It was already recognized by Westheimer [56] that these kinematic relations, known in the literature as the "main sequence" of saccades [1], reveal a nonlinearity in the saccadic system (Fig. 1(a), "NL"). Indeed, for a linear system ("L"), the movement characteristics should scale linearly with movement amplitude: mathematically, if $s_1(t)$ is the unitary saccade for a target step of one deg, then $\lambda \cdot s_1(t)$ should be the saccade trajectory for an amplitude of λ deg. In other words, the saccade duration should be the same for all saccades, the shape of their velocity profiles should be identical, and their peak eye velocities would have to increase linearly with amplitude, λ . It is important to realize that the saccade kinematic nonlinearity is not due to extraocular plant mechanics, as the pulse-step activity profiles of the extraocular motoneurons fully reflect the main sequence properties: the pulse duration matches the saccade duration, and the peak activity of the pulse saturates for large saccade amplitudes.

Oblique saccades follow the main sequence characteristics, and because the trajectories of saccades in two dimensions are approximately straight, the velocity profiles of the horizontal, $\dot{h}(t)$, and vertical, $\dot{v}(t)$, components are scaled versions of each other: $\dot{v}(t) = \tan(\Phi) \cdot \dot{h}(t)$, with Φ the vectorial saccade direction [7, 45]. This property thus defines another nonlinearity, because the peak velocity (and duration) of a given horizontal (or vertical) movement component is not fixed, but varies with

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the saccade direction (Fig. 1(c)). For example, a 10-deg horizontal saccade (\vec{s}_0) may reach a peak velocity of 350 deg/s, and a duration of, say, 45 ms. An oblique saccade in a direction of $\Phi = 60$ deg with the same horizontal displacement (\vec{s}_{60} in Fig. 1(c)) has an amplitude of 20 deg. According to the main sequence, its vectorial peak velocity, $\dot{v}_{\rm VEC} = \sqrt{\dot{h}^2 + \dot{v}^2}$, will be about 550 deg/s (i.e., less than twice the peak velocity of a 10-deg saccade), and a duration of 75 ms. The 10-deg horizontal component of this oblique saccade then also has a duration of 75 ms, and a reduced peak velocity of only 275 deg/s (halve the velocity of the total vector). This phenomenon is called "component stretching", as movement components are stretched in time to match the duration of the movement vector (Fig. 1(c)).

Nearly every neural model of the saccadic system assumes that the main sequence properties result from nonlinear local feedback circuits in the brainstem, in which socalled medium lead burst cells in the paramedian pontine reticular formation (PPRF, for horizontal saccades; [20, 24, 43]) and in the rostral interstitial nucleus of the mesencephalic longitudinal fasciculus (riMLF, for vertical-torsional saccades [21]; we ignore torsion in this paper) are thought to be driven by horizontal and vertical dynamic motor error signals, respectively (see Fig. 3).

The brainstem burst cells transform the dynamic motor-error signal into the component's eye-velocity output, a process known as pulse (or burst) generation. The far majority of saccade models assume that the input-output characteristics of the pulse generators follow a saturating nonlinearity that mimics the amplitude-peak velocity relation of Fig. 1(a) (e.g., [36, 45]). The underlying idea is that the presumed high-gain burst cells are driven into saturation for almost all saccades, and that as a result of neural fatigue this affects the shape of their instantaneous input-output relationship. Fatigue then accounts for the skewness of saccade velocity profiles: a short, nearly constant acceleration toward peak eye velocity in about 20-25 ms, followed by a saccade-amplitude-dependent deceleration phase until the saccade has ended [49]. As a result, small saccades have nearly symmetrical velocity profiles (roughly equal acceleration and deceleration phases), but large saccades are positively skewed as the deceleration phase outlasts acceleration.

However, the presumed nonlinearities of the local feedback loops lead to nasty problems for the instantaneous coupling of horizontal and vertical movement components of oblique saccades (Fig. 1(c); e.g., [17, 27, 37, 45]). Given the complications to produce straight oblique saccades, we believe the question is justified why the pulse generators would contain a saturating nonlinearity in the first place.

We argue that there is surprisingly little data to support the commonly held assumption that the input-output characteristic of the pulse generator underlies the main sequence nonlinearity. A critical problem is that the true nature and dynamics of the input signals to the brainstem burst cells (presumed to be dynamic motor error) are in fact unknown, as single-cell recordings only

provide information about the cell's output (which is strongly related to the eye velocity component). Furthermore, we believe it is unlikely that the nonlinearity of saccade kinematics would reflect a mere passive saturation, or neural fatigue, for the following reasons:

First, behavioral data have shown that the much slower saccades to remembered targets in darkness obey their own nonlinear main sequence, in which the velocity asymptote is considerably lower than for visually triggered saccades [37].

Second, neurons in the abducens and oculomotor nuclei reach firing rates that are as high as those of the brainstem burst neurons. Yet, models of the saccadic system invariantly employ linear transfer characteristics to describe these other types of neurons.

Third, even though oculomotor neurons do have nonlinear characteristics (they essentially behave as rectifiers, since they are recruited beyond a threshold), it is thought that the output of the total neuronal population varies approximately linearly with changes in eye position across the oculomotor range [44]. Taken together, although the concept of a nonlinear brainstem burst generator is a central dogma in saccadic system theories, its experimental evidence is not compelling. Further, it is questionable whether nonlinearities at the single-cell level inevitably cause nonlinearities at the population level of the brainstem burst generators (e.g., [41, 44]).

1.2. The midbrain-brainstem saccade generator

It is well established that a population of neurons in the motor map of the midbrain superior colliculus (SC) encodes the coordinates (amplitude and direction) of an upcoming saccadic eye movement (Fig. 2). The idea is that since each cell in the SC prefers a particular movement vector (quantified by its movement field; Figs. 2(a)-2(c)), the total saccade is determined by combining the weighted contributions from all recruited cells. The weights are thought to depend on two factors: (i) the fixed connection strength of a cell with the horizontal and vertical burst generators, which is completely determined by its location in the SC motor map, and (ii) the cell's activity, which depends on the intended saccade vector.

However, there is controversy as to how the SC population activity is eventually decoded/weighted by the brainstem. In particular, the discussion boils down to the question whether saccade-related SC activity also carries information regarding the eye movement trajectory and its kinematics (a *motor theory* of the SC), or whether the temporal details and levels of the activity patterns are immaterial, as the population merely encodes the saccade vector (the "goal theory" of the SC). We here compare two competing schemes from the recent literature that well illustrate this controversy, and are both supported by neurophysiological evidence: the vector averaging scheme (proponent of the goal theory) versus the vector summation model (proponent of the motor theory), respectively. We will first describe these two models in some detail, to better enable a quantitative comparison.



Fig. 2. Single-unit movement field and microstimulation in monkey superior colliculus. (a–c) Neural activity (in gray scale) for saccades into the cell's movement field (optimum at $[R, \Phi] = [15, 240]$ deg). (a) Amplitude scan. Gray eye movement trace: visual saccade into movement field center; black: saccade evoked by microstimulation at 20 μ A (500 Hz; cathodic pulses). Inset right: number of spikes (with sd) as function of amplitude. (b) Direction scan through the movement field, with eye velocity traces for visual (gray) and electrical microstimulation (black). Inset: number of spikes as function of direction. (c) Movement field. Dots: visual saccades, selected for amplitude and direction scans. Black trace: stimulation-evoked saccade. (d) Stimulation saccades and optimal visual saccades for 14 different sites. Stimulation evokes a saccade into the center of the movement field of recorded cells near the stimulation electrode.

1.2.1. Vector averaging

According to the vector-averaging hypothesis, the SC output specifies a dynamic oculocentric goal, which is computed by averaging all cell contributions from the recruited population by:

$$\vec{S}_{\text{AVG}}(t) = \frac{\sum_{n=1}^{N_A} f_n(t) \cdot \vec{R}_n}{\sum_{n=1}^{N_A} f_n(t)}.$$
(1.1)

with N_A the total number of active cells, $f_n(t)$ the instantaneous firing rate of cell n, and \vec{R}_n the fixed movement vector corresponding to the locus of cell n in the motor



Fig. 3. The SC-brainstem saccade controller. The SC motor map is logarithmically compressed in the rostral (r)–caudal (c) direction for saccade amplitude (shown from R = 5 to R = 80 deg), and is roughly linear for saccade direction (medial, upward saccades, to lateral, for downward saccades). The brainstem (right) contains dynamic feedback loops to generate horizontal/vertical saccade component velocities for clarity, only one burst generator is shown. (a) Nonlinear vector-averaging model (after [55]). RSI: resettable integrator. AVG: vector averaging of SC output. ΔE : current eye displacement. NL: dynamic brainstem nonlinearity. H, V: gain modulation of horizontal/vertical burst generators during oblique saccades; the fatigue mechanism is not included. (b) Linear vector-summation model (after [15]). D: saccade duration; v_{SAC} : current saccade velocity; G: gain of linear burst generator.

map [23, 30, 55]. Equation (1.1) effectively computes the center-of-gravity of the population in motor coordinates. The saccadic burst generator is then driven by dynamic eye motor error, $m_E(t)$, which is the difference between the SC goal (which is reached almost instantaneously) and the current eye displacement; the latter is obtained by integrating the burst-generator's velocity output (Fig. 3(a)). To account for the nonlinear saccade main sequence [1, 56], the burst generator's nonlinear input-output characteristic is usually written as:

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$$\vec{v}_{\text{SAC}}(t) = V_0 \cdot (1 - \exp(-\alpha_0 \cdot \vec{m}_E(t))),$$

where $\vec{m}_E(t) = \vec{S}_{\text{AVG}}(t) - \int_0^t \vec{v}_{\text{SAC}}(\tau) \cdot d\tau.$ (1.2)

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The integration in Eq. (1.2) yields the current eye displacement signal, and implements a so-called resettable integrator (RSI; [22]); its value is reset to zero when the saccade has finished (and the eye foveates the goal specified by the SC). Although in some models this reset is proposed to have a relatively long time constant of about 50 ms, leading to large saccade errors at very brief (< 30 ms) intersaccadic intervals [26], we here take the reset as instantaneous [13].

In Eq. (1.2) we omitted, for ease of description, the decomposition of the vectorial saccade command into its respective horizontal/vertical movement components. In the vector averaging scheme this is implemented by a complex cross-coupling scheme for the horizontal and vertical burst generators (schematically indicated in Fig. 3(a) by H, V). This cross-coupling lowers the burst gains, $\alpha_{0,H} \cdot V_{0,H}$ and $\alpha_{0,V} \cdot V_{0,V}$, to account for component stretching and straight saccades (see above; [17, 27, 45]). To account for skewness of the saccade-velocity profiles (Fig. 1(b)), the static saturating characteristic of Eq. (1.2) should be dynamic too, e.g., by making the angular constant, α_0 , time dependent. In this way, the slope of the characteristic toward saccade offset will systematically vary with the saccade amplitude.

To summarize, the averaging model of saccade generation (Fig. 3(a)) contains at least four nonlinearities: (i) vector averaging of the superior colliculus population activity; (ii) saturation of the brainstem burst generator to account for the nonlinear main sequence; (iii) fatigue of brainstem burst cells to yield appropriately skewed velocity profiles; (iv) cross-coupling between horizontal and vertical burst generators to explain straight oblique saccades and component stretching.

1.2.2. Vector summation

We recently proposed an alternative encoding scheme with only linear operations that accounts for the main sequence kinematics, and straight oblique saccades ([15], Fig. 3(b)). In this linear model, the SC motor map directly encodes the desired saccade trajectory through dynamic *linear vector summation* of its cell contributions:

$$\vec{S}_{\text{LIN}}(t) = \sum_{n=1}^{N_A} \sum_{k=1}^{N_S} \vec{s}_n \cdot \delta(t+20-\tau_{k,n}), \qquad (1.3)$$

with N_S the number of spikes of SC cell n (counted from 20 ms before saccade onset to 20 ms before saccade offset; the 20 ms being the SC-lead time); $\delta(x)$ is the Dirac impulse function (i.e., a spike); $\tau_{k,n}$ is the time of the k-th counted spike of cell $n; \vec{s}_n = \eta \cdot \vec{R}_n$ is the so-called "spike-vector" of cell n, which is the (tiny) eyedisplacement vector generated by a single spike of cell n, where η (~ 1e-04) is a fixed scaling constant, which is identical for all cells [15, 48].

In this scheme (Fig. 3(b)), the burst generator is driven within a linear feedback circuit by motor error, which is specified by the integrated difference between the

instantaneous SC output, and the burst generator's velocity output (reminiscent to Scudder's model, [36])

$$\vec{v}_{\text{SAC}}(t) = G \cdot \vec{m}_E(t),$$

with $\vec{m}_E(t) = \int_0^t \left[\left(\sum_{n=1}^{N_A} f_n(\tau) \cdot \vec{s}_n \right) - \vec{v}_{\text{SAC}}(\tau) \right] \cdot d\tau.$ (1.4)

G is the (fixed) gain of the burst generator (i.e., no cross-coupling), and $f_n(t)$ is the instantaneous firing rate of cell n in the SC motor map (note that integration of the SC-firing rate over time is the same as computing the cumulative number of spikes, as in Eq. (1.3)). For simplicity, we here left out a small delay in the local feedback loop. Note that, according to this simple model, the instantaneous collicular output specifies the instantaneous eye velocity vector by:

$$\vec{v}_{SAC}(t) = \frac{G}{1+G} \cdot \sum_{n=1}^{NA} f_n(t) \cdot \vec{s}_n.$$
 (1.5)

Simulations that used measured spike trains from about 150 recorded neurons demonstrated that this simple model (only two free parameters: gain G and feedback delay τ) generated saccades with the correct trajectories, velocity profiles, and main-sequence behavior [15]. Further, the model makes two quantitative predictions for SC spike trains that were experimentally verified for the entire population of recorded cells: (i) the cumulative number of spikes of each recruited neuron and the instantaneous straight-line displacement of the eye between start and end position are linearly related, and (ii) the cumulative number of spikes, N(t), is predicted by a precisely defined, dynamic extension of the classical movement field of Ottes *et al.* [29] (see [15], for details)

$$N(t) = \frac{N(R, \Phi)}{R} \cdot \Delta \hat{e}(t - 20), \qquad (1.6)$$

with $N(R, \Phi)$ the movement-field description for the total number of spikes in the burst ([29]; see the insets of Figs. 2(a) and 2(b) for an example), $[R, \Phi]$ saccade amplitude and direction, and $\Delta \hat{e}(t)$ the instantaneous, straight eye-displacement between the on- and offset positions (delayed by 20 ms).

1.2.3. Main sequence

Because the brainstem circuitry in the vector summation model is described by a linear feedback system, the nonlinear behavior of the saccade kinematics, the shape of the velocity profiles, and component stretching, should all be embedded in the spatio-temporal activity patterns of the SC motor map. The model and data described so far (and in [15]) do not specify how the population activity could encode these kinematic properties.

To study this point, we therefore first resorted to simulations with a computer model of the SC-brainstem system, in which we investigated the effect of different



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Fig. 4. Normalized burst profiles for SC cells at four different rostral-caudal locations have different skewness and burst durations. The number of spikes in these bursts, however, is highly similar. Small (6 deg) saccades have symmetrical bursts, those of large (32 deg) saccades are highly skewed. Insets show the associated average saccade position (top) and velocity (bottom) traces (adapted from [48]).

potential activity patterns [48]. These simulations used the well-established idea that static SC movement fields (quantified by the total number of spikes in the burst for a given saccade) are described by a spatial Gaussian activity profile in the SC motor map, with a width, $\sigma_{\rm pop} \approx 0.5$ mm, that is invariant across the SC [29]. Hence, with the assumption that the SC cell density is constant too, any given saccade recruits the same number of cells in the motor map.

We approximated the *temporal* profile of saccade-related bursts of each recruited cell in the population (illustrated in Fig. 4) by an asymmetric gamma-function:

$$f_n(t) = \kappa \cdot t^{\gamma} \cdot \exp\left(-\frac{t}{\beta}\right), \qquad (1.7)$$

in which γ (dimensionless) is a measure for the burst skewness, and β (in ms) a parameter that sets the burst duration (Fig. 5(b)). The peak of the burst is determined by κ , and depends on the distance of the cell from the population center (in other words, on the distance between the cell's optimal saccade vector, $[R_n, \Phi_n]$ and the actual saccade for which it is recruited; see e.g. Fig. 2). The cell's peak firing rate is therefore determined by $F_{\rm pk} \cdot \kappa$, with $F_{\rm pk}$ the peak firing rate at the center of the population. Note that in this formulation, all cells in the population have synchronized bursts.

So how does such a population generate the main-sequence kinematics? The simulations with the vector summation model showed that if the burst parameters $[F_{\rm pk}, \gamma, \beta]$ are taken constant across the motor map (i.e., the same for all cells), the



Fig. 5. Simulated burst properties of saccade-related cells in the SC motor map. (a) For all cells, the total number of spikes for their optimal saccade (Nspikes) is fixed; burst duration (BurstDur) and burst skewness both increase with the optimal saccade amplitude; peak firing rate (PeakFR) drops with amplitude, as $F_{pk} \propto 1/\sqrt{R_{\text{opt}}}$. (b) Burst profiles for locations at the rostral-to-caudal extent of the motor map (Eq. (1.7)). Compare with Fig. 4 (adapted from [48]).

total system behaves as a *linear* system: saccades will have fixed durations and linearly increasing peak velocities (like in Fig. 1(a), "L"). The only way to obtain the main sequence was by assuming that the burst profiles of Eq. (1.6) are not invariant across the map. The nonlinear main sequence of saccades (Fig. 1(a)) emerges when the firing rates of SC cells vary in a systematic way with their rostral-to-caudal location within the SC motor map: cells at the rostral zone (r in Fig. 3), encoding small saccades (i.e., they generate small spike vectors) have high peak firing rates (up to 700-800 Hz), short burst durations, and near-symmetrical burst profiles (large γ ; see Fig. 4 for experimental evidence for this assumption). In contrast, at the caudal end (c) of the map, where large saccades are encoded (Fig. 3), peak firing rates are lower (down to 300-400 Hz), burst durations are longer, and burst profiles are positively skewed (low γ , [16, 48]; Fig. 5(b)).

In line with the recordings (Fig. 4), the total number of spikes in the burst was taken fixed across the motor map (at about 20 spikes for the central cell of the population), despite differences in the burst shapes ([15, 48]; Figs. 4 and 5), and strongly varying kinematics during blink-perturbed responses [14, 15]. Finally, the horizontal and vertical components of an oblique saccade in direction Φ are determined by linear vector decomposition of the radial velocity command (Eq. (1.5)), by scaling through synaptic weightings representing $\cos(\Phi)$ and $\sin(\Phi)$, respectively (i.e., no cross-coupling needed).

1.2.4. Why vector averaging?

The vector averaging scheme of the SC was first proposed by [23] to explain the pattern of changes of saccade endpoints after inducing a local, reversible

inactivation in the SC motor map. Interestingly, this subtle lesion affected the saccade metrics (i.e., their amplitude and direction) in a way that could not be predicted by linear vector summation, but was nicely in line with vector averaging. All saccade endpoints appeared to be directed away from the lesion: only saccades into the center of the inactivated spot were accurate; saccades to a site caudal (or rostral) from the lesioned site were hypermetric (or hypometric). Similar effects were seen for saccades with different directions. However, the inactivation also affected the saccade kinematics, as all post-inactivation saccades were slower (by about 20%) than control saccades of the same amplitude. The latter effect was not immediately predicted by vector averaging, as the averaging scheme (Eq. (1.1)) only specifies the saccade metrics. To account for the effect on kinematics, the authors assumed that somehow the SC firing rates influenced saccade velocity, presumably by affecting the gain (asymptote) of the brainstem burst generator [23, 27, 55] (Fig. 3(a)).

However, Goossens and Van Opstal [15] demonstrated that the same effects on saccade metrics and kinematics emerge from the linear vector summation model of Eqs. (1.3) and (1.5), provided one feature was added: the saccade would stop after reaching a fixed total spike-count criterion (i.e., it necessitates an active saccade-offset mechanism). In this way, when the center of the population is silenced by subtle chemical inactivation, the remaining active cells along the fringes of the Gaussian population, which have lower firing rates, will (i) produce a slower saccade, and (ii) will have to fire longer in order to reach the required total spike-count. Simulations of the summation model with real measured spike trains from about 150 SC neurons showed that the Lee *et al.* (1988) data on saccade metrics and kinematics could be reliably reproduced with this spike-count mechanism, which in fact introduces a brainstem nonlinearity. The recordings further supported the idea that recruited SC populations may indeed produce an invariant number of spikes for normal saccades across the oculomotor range [15].

1.3. Target selection: Averaging and updating

1.3.1. Double stimulation

When the visuomotor system is confronted with two visual stimuli presented simultaneously at different retinal locations, say at \vec{T}_1 and \vec{T}_2 , it often programs an eye movement that lands at a location between the two stimuli [8, 28]. Since this response resembles the center-of-gravity of the retinal input vectors, its endpoint is usually described by a weighted vector-averaging response:

$$\vec{S}_{\text{AVG}} = \frac{a \cdot \vec{T}_1 + b \cdot \vec{T}_2}{a + b}.$$
(1.8)

Here, a, b are fixed target weights that may depend on a number of different factors, like their relative saliencies and size [8], their spatial-temporal separation [2, 28], task constraints (e.g., target versus non-target), or on the saccade reaction time

[2, 28]. Because of the nonlinear scaling (division by a + b), Eq. (1.8) implies the existence of yet another nonlinearity in the visuomotor transformation underlying saccade programming.

Several studies have supported the idea that target averaging could be the underlying mechanism for dealing with multiple stimuli, and neurophysiological studies have suggested that such averaging occurs downstream from the SC motor map, in line with the assumptions of Eq. (1.1) (Fig. 3(a)). For example, electrical microstimulation at two locations within the SC motor map produces a saccade that resembles the weighted vector average of the two sites stimulated separately [31]. Recording studies have suggested that also under natural visual double-stimulation conditions each of the two visual stimuli induces an active population at the respective retinal sites in the motor map ([5], for express-latency saccades, and [30]).

It may be noted that the Edelman and Keller data [5] indicated that the peak activity at the two retinal target representations for express saccades was substantially lower than for the single-target control stimuli. This hints at the possibility that long-range inhibitory interactions ("winner-take-all") may govern the competition between different active populations within the SC motor map, a mechanism incorporated by [51] in a modification of the original ensemble-coding model [46] to explain Robinson's electrical double-stimulation results [31]. In this way, linear vector addition could in principle still account for presumed "downstream averaging", as in that case the summed output of each stimulated population (Eq. (1.3)) would be reduced by the lower firing rates.

Unfortunately, the exact spatial-temporal distribution of activity in the SC motor map under multiple-target conditions (knowledge that is crucial to apply the summation model; see below) is not well established. For example, other experiments had shown a single-peaked population in the SC motor map at the averaging site represents the averaging saccade in a double-stimulus saccade paradigm ([5], for regular-latency saccades; [11, 50]). These data therefore suggested that target averaging could have occurred *upstream* from the motor SC.

1.3.2. Double-step paradigm

When the two targets are presented in rapid temporal succession in a double-step, in which the first target rapidly changes direction (after a brief delay, Δt) toward a new location, strongly curved saccade responses may be observed [30, 47]. In the framework of Eq. (1.8), the double-step would involve a delayed activity at the retinal representation of target $T_2: T_2(\Delta t)$.

The problem with the model of Eq. (1.8), also acknowledged by [30], is that it fails to account for experimentally observed curved trajectories in the double-step task. Such trajectories may start off in any initial direction, to eventually curve toward the final goal (see e.g., Fig. 6(b)). Not only does the averaging saccade of Eq. (1.8) have a wrong end-direction (note that its end-direction will be fully determined by the remaining activity at site T_2 , and hence be parallel to the *retinal vector* T_2), it



Fig. 6. (a) Linear vector computations could underlie saccade averaging (Eq. (2.1)), and (b) curved saccades (Eq. (2.2)). (c) An identical target configuration can thus give rise to a myriad of differently curved saccades and, similarly, (d) many different underlying motor commands for S_{AVG} and S_2 could result in very similar, curved trajectories (see text, for explanation).

also cannot reach the final end point at T_2 once the saccade has started off into a different initial direction.

A solution to this problem was incorporated in an improved version of the conceptual vector-averaging model by [55] (Fig. 3(a)) who simulated weighted vectoraveraging saccades and curved trajectories. In the Walton model, the constant weights of Eq. (1.8) were replaced by time-varying weights at the two sites that represented the population activities:

$$\vec{S}_{AVG}(t) = \frac{w_1(t) \cdot \vec{T}_1 + w_2(t) \cdot \vec{T}_2}{w_1(t) + w_2(t)}.$$
(1.9)

The vector-averaging model of Walton *et al.* ([55]; Eqs. (1.1) and (1.9)) assumes that the two retinal targets appear as consecutive active populations in the SC motor

map, and that the activity associated with the second target outlasts the first target. Accordingly, the "center-of-gravity goal" for the saccade will gradually shift during the movement from T_1 to T_2 , and the saccade will curve in midflight to follow the dynamic goal, which eventually settles at the second retinocentric target location [55].

1.3.3. Spatial updating

So far, the described modeling efforts of target selection mechanisms did not incorporate the important concept of spatial updating within the visuomotor system. Robinson proposed that the saccadic system cannot rely exclusively on the retinal coordinates of a target to program its oculomotor response under dynamic conditions, and that it therefore employs feedback about its own oculomotor behavior (through efference copies of motor commands) to ensure spatial accuracy [32]. Behavioral double-step experiments showed that the saccadic system accounts for intervening eye movements to update the coordinates of briefly flashed visual stimuli in darkness [18], even when flashed in midflight of an ongoing headunrestrained saccade [53].

In addition, electrophysiological studies had indicated that SC cells faithfully reflect the oculocentric target location (i.e., the oculomotor error), rather than the retinal locus of a flashed target, after an intervening saccadic eye movement. Interestingly, this was even the case when the saccade was not planned by the monkey, but evoked by the experimenter through SC micro-stimulation [38], but compensation for an electrically evoked eye movement did not occur when the stimulation was applied to the brainstem burst generator [39]. Those seminal experiments demonstrated that activity in the SC motor map corresponded to the true oculomotor error, required to direct gaze to the extinguished target location, and provided strong evidence that spatial updating for saccades employed an efferent feedback signal that occurred at or upstream from the recorded cells. Subsequent recordings in the frontal eye fields suggested that the visuomotor system keeps visual targets in an eye-centered reference frame by accounting for changes in eye position induced by intervening saccades [12].

An elegant series of neurophysiological experiments in posterior parietal cortex [4], frontal eye fields [42], and in midbrain SC [54], have later demonstrated that responses of many cells even *predict* the visual consequences of an upcoming, *planned* saccade. Hence, when the monkey programs a saccade, a cell in either of these areas signals an updated visual response *before* that saccade is even initiated, a phenomenon that was termed *predictive remapping*.

Here we speculate that predictive remapping, as well as stimulation-evoked updating (necessarily occurring during or after the evoked saccade), may use the same remapping mechanisms.

In this paper, we explore how spatial updating might be incorporated in the framework of the vector-summation model, by showing that target averaging and the programming of goal-directed curved saccades in the double step

paradigm may in fact be understood from linear vector operations, in combination with predictive remapping.

2. Methods and Results

2.1. Oculocentric remapping revisited

When the saccadic system responds with two saccadic eye movements to a doublestep target jump, the respective oculomotor error signals are determined by the first retinal target location, T_1 , and by the difference between the retinal stimuli, respectively:

$$\vec{T}_1$$
 and $\Delta \vec{T} = \vec{T}_2 - \vec{T}_1.$ (2.1)

Since predictive remapping holds that the second signal is already available to the system before the first saccade is executed (see Introduction), we here note that the "averaging" response of Eq. (1.8) could then also be formulated as the linear vector sum of these two signals:

$$\vec{S}_{AVG} = \vec{T}_1 + \alpha \cdot \Delta \vec{T}$$
 in which $0 \le \alpha \le 1$. (2.2)

When $\alpha = 0$ the first saccade is directed toward T_1 , for $\alpha = 1$ it is directed toward T_2 , whereas for $\alpha = 0.5$ it goes towards the midpoint (Fig. 6(a)). Thus, the coordinates of the averaging saccade could simply be determined by weighted linear vector addition, without the need for a nonlinear scaling mechanism.

Conceptually, parameter α could correspond to the relative activity level of the population associated with the developing difference vector. It could vary from trial to trial, and may e.g., depend on visual stimulus properties (stimulus timing, intensity, size), or on the latency of the first saccade. However, regardless of the mechanism that modulates α , the point is that experiments have shown that the coordinates of the difference vector are already available before the start of the first saccade.

A complete linear parameterization of curved saccades can also account for the possibility that the first response is not directed toward T_1 , but is an averaging response that may only be partially executed (amplitude gain β , with $0 < \beta < 1$). Behavioral experiments have shown that the saccadic system indeed accounts for the trial-to-trial variability in its own motor performance [13, 53], so that the programming of the second saccade vector incorporates the coordinates of the actual first saccade, S_{AVG} , rather than the retinal target coordinates of T_1 and T_2 .

In our extended vector-summation model, a curved saccade is thus determined by the linear sum of two overlapping motor commands, in which the second, delayed, command accounts for the metrics of the first response:

$$\vec{S}_{\text{SUM}}(t) = \vec{S}_{\text{AVG}}(t) + \vec{S}_2(t-\tau), \quad \text{with } \vec{S}_2(t) = \vec{T}_2 - \vec{S}_{\text{AVG}}(t) \quad \text{and}$$

$$\vec{S}_{\text{AVG}} = \beta \cdot (\vec{T}_1 + \alpha \cdot \Delta \vec{T}).$$
(2.3)

Equation (2.3) contains three free parameters, which give rise to a myriad of curved responses for an identical target configuration: α (the amount of "averaging", Eq. (2.2)), β (the gain of the averaging saccade), and delay τ . Note that if $\beta = 1$, the second saccade is simply determined by $\vec{S}_2 = (1 - \alpha) \cdot \Delta \vec{T}$.

Figures 6(c) and 6(d) show a number of examples that were simulated according to Eq. (2.3), using simplified Gaussian velocity profiles. In Fig. 6(c) we generated trajectories for a variety of parameters (black traces: $0 \le \alpha \le 0.9$, $\beta = 1$, and $\tau = 70$ ms, showing various amounts of averaging; magenta and green traces: $0 \le \alpha \le 0.9$, $\tau = 70$ ms, and $\beta = 0.75$, versus 0.50; blue traces: $\alpha = 1$, $\beta = 1$ and $\tau = 15$, 30, 100, 150 and 250 ms. In the last case, the first saccade briefly stopped at T_1 , before continuing with the difference response).

Figure 6(d) illustrates the point that many different combinations of S_{AVG} and S_2 can give rise to highly similar trajectories. In this simulation, the black "target trace" served as a measured template ($\alpha = 0.4$, $\beta = 0.7$ and $\tau = 35$ ms) for which the underlying motor commands were subsequently estimated by a brute-force search across 8000 different [α, β, τ] combinations. The magenta traces are solutions that differ by less than 2% from the target trajectory (considered to fall within the measurement resolution). The green and blue examples highlight two very different solutions for the two motor commands that yet yielded nearly identical trajectories.

This point therefore illustrates an important problem, namely that if vector summation were to be the mechanism underlying saccade updating, SC recordings at one or two sites will not suffice to unravel the two underlying motor commands, as they could vary tremendously from trial to trial, without affecting the emerging trajectories.

Figure 7 illustrates a full simulation of a curved saccade response by the vectorsummation model, according to Eqs. (1.5) and (2.3). The MatLab implementation of this model is the same as described in [48].

In this simulation, the two targets at $[R, \Phi] = [20, \pm 45]$ deg induced a brief visual response in the visuomotor cells, followed by motor activity, which is relocated to two different sites within the SC, given by $\alpha = 0.4$, $\beta = 0.9$, and a delay $\tau = 30$ ms.

2.2. Curvature in single-target responses

In the vector-averaging model, the nonlinear horizontal and vertical burst generators are coupled through mutual inhibition to reduce the gains of the burst generators in a way that depends on amplitude and velocity of each response component. This additional nonlinearity is introduced so as to produce straight oblique saccades. Because the burst generators have linear characteristics in the vector-summation model, such a precisely tuned coupling is not required. In this model, the burst generators are independent, but saccades are straight because (i) the cells in the SC population fire roughly synchronously [15, 16], and (ii) the gains and delays in the local feedback loops are taken identical.



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Fig. 7. Simulation of a curved saccadic response by the output of the motor SC. The curved saccade consists of an initial averaging saccade, temporally overlapping with a second saccade toward the final goal (with $\alpha = 0.6$, $\beta = 0.9$, and delay, $\tau = 30 \text{ ms}$). The visual activity of the SC cells (a-b) arises at the retinal representations of the target locations, at two different moments in time, 30 ms apart. The motor map (c-d) shows the initial motor response at t = 130 ms toward the averaged location (c), followed by motor activity at the right-downward representation, here shown at t = 160 ms (d). In (e), the resulting saccade trajectory (red trace) is shown; the blue dashed lines correspond to the two underlying individual saccade vectors S_{AVG} and S_2 , respectively. Panel (f) shows the horizontal/vertical eye positions and corresponding radial eye velocity as a function of time.

Notwithstanding, this model also can readily produce curved saccades toward single targets. For example, when the BG gains differ, both burst generators produce different component kinematics, and saccade trajectories curve toward the faster component. A simulation of Niemann-Pick Type C disease [33] with the linear summation model provides an extreme example of this uncoupled component kinematics (Fig. 8). In this simulation, the gains of the burst generator feedback loops were set to $B_H = 80$ and $B_V = 8$, respectively.

Note that another mechanism by which the SC population could in principle produce a curved saccade would be by desynchronizing activity among the cells within the neural population. However, our recordings indicated that for saccades evoked by a single target this simply does not happen, as the saccade-related bursts within the active population appear to be highly synchronized [16].

In summary, according to the vector-summation model, nonlinearities in the spatial processing within the saccadic system ("target averaging", and "curved double-step responses") could result from linear vector processing through oculocentric spatial updating (predictive remapping), whereas the nonlinear saccade kinematics ("main sequence", "skewness", "component stretching") could all result from a rostral-to-caudal shaping of the SC burst profiles, followed by dynamic linear addition of all cell contributions at the brainstem burst generators.



Fig. 8. Simulation of Niemann Pick Type C disease, for which failure of the vertical burst generator is assumed. This was simulated by setting the gains of the burst generators for horizontal and vertical saccade components to $G_H = 80$ and $G_V = 8$, respectively. Feedback delays are 4 ms. Insets: (b) patient data, and (a) normal oblique saccades (from [33], with permission).

3. Discussion

3.1. The SC as optimal controller

This paper compares two conceptual models that propose how the midbrain SC is involved in the generation of a saccadic eye movement: the vector-averaging model versus the vector summation model. Both models can account equally well for a myriad of well-known saccade behaviors: the nonlinear kinematics of the main sequence, the skewness of saccade velocity profiles, component stretching in oblique saccades, and target averaging. Note also that both models place the motor SC outside the local feedback loop of saccade generation: both rely on the experimental finding that the population of recruited cells for a saccade stays at a fixed location in the motor map (i.e., no moving population). However, the models disagree about the role of the SC in specifying the saccade kinematics. In the vector summation model, the SC output represents a dynamic, desired eye-displacement signal (i.e., a feedforward motor command signal), represented by the cumulative number of spikes in the bursts of all recruited cells; the vector-averaging scheme provides a gaze-motor error signal (the saccade goal) to the brainstem.

Although the mean level of SC activity has been proposed before to influence saccade kinematics, e.g., by affecting the characteristics of the brainstem burst

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generator [27, 40, 55], the precise mechanism of this putative pathway has remained elusive, and at best, speculative. Similar problems haunt the neurocomputational implementation of the vector-averaging process: although mathematically straightforward (Eq. (1.1)), it remains unclear and highly nontrivial how downstream averaging of cell contributions is carried out by neurons at the midbrainbrainstem interface.

In contrast, despite its utmost simplicity (only two free parameters to predict the kinematics of all saccades), the observed saccade nonlinearities are emerging properties of the linear summation model, without additional assumptions. Moreover, the measured spike trains of saccade-related bursts in the SC follow the predictions of the spike-count model with high accuracy [15, 48].

Because the spike-summation model assumed (for the sake of argument) a *linear* brainstem circuitry, the observed nonlinearities in saccade behavior should be somehow embedded in the spatio-temporal activity patterns of the motor SC. A theoretical analysis, combined with experimental support, indicated that the main-sequence nonlinearity and velocity skewness are due to a spatial gradient across the rostral-to-caudal (i.e., amplitude) dimension of the motor map. Rostral cells fire high frequency, short, and symmetric bursts for their optimal saccade, whereas caudal cells produce lower firing rates, and longer-duration, skewed burst profiles. These properties strongly suggest that the main sequence of saccades is not due to a passive nonlinear characteristic in the brainstem (like neural fatigue, and saturation), but rather implement a *deliberate neural control strategy*.

Given the function of saccades (redirect the fovea as fast and as accurately as possible to the target), in combination with internal constraints (neural noise, poor spatial resolution in the peripheral retina), Harris and Wolpert demonstrated on the basis of a theoretical analysis that the main sequence is an optimal solution in the sense of speed-accuracy trade off [19]. As the analysis was applied to horizontal saccades only, it could not dissociate brainstem nonlinearities from collicular nonlinearities. However, when considering two- (or even three-) dimensional control of eye movements, or combined eye-head gaze shifts, the motor SC is ideally suited to implement such an optimal control strategy. With a single, vectorial nonlinear burst generator, implemented in the SC motor map, the main sequence is guaranteed for all saccade directions, for which straight oblique saccades are a mere byproduct, without the need for complex, dynamic cross-coupling of the horizontal and vertical nonlinear burst generators.

Note that in the vector summation model, the saturation of peak-eye velocity in the main sequence for large saccades results from two opposing factors: (i) lower firing rates at caudal sites, which roughly follows the inverse of the square-root of saccade amplitude (see [48]), versus (ii) the exponential increase of synaptic efficacy with the SC rostral-caudal coordinate ($w \propto \exp(u)$, with u running from 0 (fixation area) to about 4 mm) according to the efferent mapping function [29]. If peak-firing rates at rostral and caudal cells would be the same, peak eye velocity would increase linearly with saccade amplitude. Since we see no obvious physiological reason as to

why caudal cells would fire at lower rates than rostral cells, we conjecture that this gradient implements a deliberate, functionally relevant, design within the gaze-control system.

This putative role in optimal motor control is obviously not in line with the goalhypothesis for the SC output population. Indeed, it is difficult to reconcile the tight correlations of instantaneous firing patterns of SC cells with the ongoing eye-movement kinematics (Eqs. (1.5) and (1.6)), and the rostral-caudal gradient of burst properties (Fig. 4), without accepting an explicit role for these cells in oculomotor control.

3.2. Averaging mechanisms

Several phenomena could resemble "vector averaging", that may nonetheless result from quite different mechanisms. The weighted vector-averaging operations of Eqs. (1.1), (1.8) and (1.9) produce the center-of-gravity of the contributing cells in motor (or visual) space. The same holds for the linear vector operation described by Eq. (2.2). Indeed, the two are equivalent in their predictions, although the underlying mathematics (nonlinear scaling, versus linear vector addition) is quite different.

However, there is a third way in which neural activity of the SC motor map could produce responses that may resemble a weighted average. For example, an intrinsic local excitatory-global inhibitory network between SC cells (e.g., a Mexican hat connectivity pattern) would lead to competition between distant neural populations, e.g., at the neural images of T_1 and T_2 , that either produces a winnertakes-all activation at one of the sites, or to reduced activities at either site. Such a mechanism operates in collicular space, and although the result will strongly deviate from the linear addition of the vectors corresponding to T_1 and T_2 , it is not a proper vector average either (e.g., [51]). This third mechanism could underlie the results observed for synchronous electrical double stimulation at different collicular sites [31], and may also explain the observed neural responses for saccades evoked at express latencies by visual double stimuli [5]. In the latter case, the visual responses at the original retinal locations are thought to merge with the short-latency motor activities, which then would interact through the Mexican hat connections. For longer reaction times, however, predictive remapping (Eq. (2.2)), or proper averaging at an upstream processing level (e.g., in frontal eye fields, or posterior parietal cortex), could relocate the neural activity in the SC motor map to the appropriate motor representation (as illustrated in Fig. 7).

3.3. Comparison to the skeletal motor system

Cells in the primary motor cortex (M1) of primates are tuned for arm-movement direction, and the "*population-vector*" hypothesis [9, 10] describes how the weighted contributions of M1 cells encode the arm-movement direction by:

$$\vec{P}_{\rm arm} = \sum_{n=1}^{N_A} f_n \cdot \cos(\phi_n) \cdot \hat{e}_n, \qquad (3.1)$$

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with f_n a neuron's firing rate, and φ_n , the arm-movement angle re. its preferred unity direction vector, \hat{e}_n .

Note that the spatial tuning of M1 cells differs fundamentally from the movement-field tuning of SC cells: first, the directional tuning of SC cells is restricted to about 60 deg, and is therefore much narrower than the cosine tuning of M1 cells (see e.g., Fig. 2(b); second, unlike SC cells, responses in M1 do not appear to be restricted for movement amplitude. In that respect, the spatial tuning of M1 cells is reminiscent to the movement fields of brainstem burst cells in PPRF and riMLF, which are also cosine tuned (Fig. 3). A theoretical analysis demonstrated that a uniform population of independent cosine-tuned cells always provides an accurate estimate of the effector's movement direction, and that therefore the population vector concept just reformulates that neurophysiological fact [34]. However, Scott et al. demonstrated that the distribution of optimal movement directions, \hat{e}_n , of M1 cells was not uniform, and that the reconstructed population-vector thus systematically deviated from the actual direction of hand movements in the horizontal plane [35]. In other words, M1 activity appeared not to encode movement within a hand-based reference frame. Instead, a better model to explain the tuning characteristics of M1 cells accounted for mechanical anisotropies of the shoulder joint (quantified by joint power) for different movement directions.

In line with the conclusions of [35] for monkey M1, local activation and stimulation studies performed in cat motor cortex also suggested a strong relationship between evoked M1 activity and selective muscle synergies [3]. For example, iontophoretically administered bicuculline (a GABA_A antagonist) in the cat's forelimb area led to a limited, but extended, spread of neural excitation of about 7 mm^2 around the injection site. The estimated size of the activated population (standard deviation about 0.7 mm) is similar to that reported for electrical microstimulation in the midbrain SC [15, 25, 29] (s.d. 0.5-0.7 mm), although the spatial neuro-anatomical connectivity in both structures is much more extensive. The limited neural excitation may be understood from the normalizing effects of a local excitatory–global inhibitory reciprocal network (see also above), effectively tuning the neural population [3, 51].

Interestingly, when applying simultaneous electrical double stimulation at different sites within the cat's motor cortex, Ethier *et al.* reported excellent correspondence with the prediction of linear vector summation for the directions of the individual stimulation effects on evoked forelimb movements [6]. Yet, the magnitude of the double-stimulation movement was typically smaller than predicted by linear vector summation. These results are qualitatively in line with similar "averaging" effects of saccadic eye movements, observed for double stimulation in the monkey SC [31]. As explained above, such averaging is probably caused by neural interactions within the intrinsic neural network between the two activated sites, rather than by a vector averaging mechanism operating in motor space. Without the assumption of Mexican hat-like connectivity within the SC, the summation model of Eq. (1.3) produces linear vector summation of the effects of

individual sites. However, as the saccade is terminated whenever the total spike count reaches the preset, fixed, population threshold, the motor output will still be restricted to the approximate "vector average" when both sites are stimulated above threshold, as in that case:

$$\vec{S}_{\rm SUM} = p \cdot \vec{S}_1 + q \cdot \vec{S}_2, \tag{3.2}$$

where p and q are the relative stimulation strengths, and the constraint p + q = 1 sets the spike-count cut-off (compare with Eqs. (2.1) and (2.2)). Nonetheless, linearity will be preserved at low stimulation strengths, when either site produces a saccade that is much smaller than the optimal vector for that site (p + q < 1) [51, 52]. Under those conditions, the linear vector sum still remains below the cut-off threshold for saccade termination. Whether such effects indeed occur for double stimulation within the SC motor map and the primary motor cortex remains to be tested.

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